Comparing and cross-validating of results from LatentGOLD Choice

It is possible to estimate μRRM models in LatentGOLD (LG) Choice 5.1 . Its implementation is however somewhat different from the implementation described in Cranenburgh et al. 2015 (henceforth referred to as the standard implementation).

Equation 1 gives the implementation of the μRRM model in LG Choice 5.1 (see Vermunt and Magidson (2014)); equation 2 gives the standard implementation.

$$R_i = \sum_{j \neq i} \sum_m \left(1 + \exp\left(\beta_m^* \left[x_{jm} - x_{im} \right]\right)\right) \qquad P_i = \frac{e^{-e^{\mu^*} R_i}}{\sum_J e^{-e^{\mu^*} R_j}} \tag{1}$$

$$R_i = \sum_{j \neq i} \sum_m \left(1 + \exp\left(\frac{\beta_m}{\mu} \left[x_{jm} - x_{im} \right]\right)\right) \qquad P_i = \frac{e^{-\mu R_i}}{\sum_J e^{-\mu R_j}} \tag{2}$$

Hence, the implementation LG Choice 5.1 differs from the standard implementation in two important ways:

1. the taste parameter β_m^* is not divided by the scale parameter μ.
2. the log of the scale is estimated, instead of the scale.

As a consequence, we cannot directly compare the results obtained using LG Choice 5.1 with the results obtained using the standard implementations. Below we show how to compare and cross-validate results obtained using LG Choice 5.1 with those obtained using the standard implementation.

Point estimates

To compare and cross-validate the point estimates of LG Choice 5.1 we can simply multiply the taste parameter estimates with the e^{μ^*} (the scale), see equation 3, where * denotes the LG parameter estimate.

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_M \\ \hat{\mu}_1 \end{bmatrix} = \begin{bmatrix} e^{\mu^*}\beta_1^* \\ e^{\mu^*}\beta_2^* \\ \vdots \\ e^{\mu^*}\beta_M^* \\ e^{\mu^*} \end{bmatrix} \quad (3)$$

Standard errors

To attain the standard error we apply the delta method (equation 4).

$$\operatorname{cov}(\Phi) = \Phi'^{T}\,\Omega\Phi', \quad (4)$$

where Ω denotes the AVC matrix of the LG parameter, Φ the vector of standard parameters, and Φ' the matrix of first derivatives of the standard parameters towards the LG parameters. Given the implementation of the µRRM model in LG Choice we get:

$$\Phi = \left[\exp(\mu^*)\beta_1^* \quad \exp(\mu^*)\beta_2^* \quad \cdots \quad \exp(\mu^*)\beta_M^* \quad \exp(\mu^*)\right] \qquad \Rightarrow$$

$$\Phi' = \begin{bmatrix} e^{\mu^*} & 0 & \cdots & 0 & 0 \\ 0 & e^{\mu^*} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & e^{\mu^*} & 0 \\ e^{\mu^*}\beta_1^* & e^{\mu^*}\beta_2^* & \cdots & e^{\mu^*}\beta_M^* & e^{\mu^*} \end{bmatrix} \qquad \Phi'^{T} = \begin{bmatrix} e^{\mu^*} & 0 & \cdots & 0 & e^{\mu^*}\beta_1^* \\ 0 & e^{\mu^*} & \cdots & 0 & e^{\mu^*}\beta_2^* \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & e^{\mu^*} & e^{\mu^*}\beta_M^* \\ 0 & 0 & \cdots & 0 & e^{\mu^*} \end{bmatrix}$$

The standard errors are the square root of the diagonal element of the covariance matrix:

$$\begin{bmatrix} SE_{\beta 1} \\ SE_{\beta 2} \\ \vdots \\ SE_{\beta M} \\ SE_{\mu} \end{bmatrix} = \begin{bmatrix} \sqrt{\mathrm{cov}_{11}} \\ \sqrt{\mathrm{cov}_{22}} \\ \vdots \\ \sqrt{\mathrm{cov}_{MM}} \\ \sqrt{\mathrm{cov}_{M+1,M+1}} \end{bmatrix}$$

Application to the shopping choice data:

The 1-class model yields the following parameter estimates, and covariance matrix Ω.

	Est	*SE*
FSG^*	0.9405	0.1996
FSO^*	–0.0096	0.0087
TT^*	–0.0865	0.0195
μ^*	–1.9711	0.2024

$$\Omega = \begin{bmatrix} 0.0399 & 0.0004 & -0.0028 & -0.0370 \\ 0.0004 & 0.0001 & -0.0001 & -0.0008 \\ -0.0028 & -0.0001 & 0.0004 & 0.0031 \\ -0.0370 & -0.0008 & 0.0031 & 0.0410 \end{bmatrix}$$

To attain the point estimates we apply equation 3. As expected, these point estimates correspond exactly with those obtained using the standard implementation, see Cranenburgh et al. 2015.

	Est
FSG^*	0.1310
FSO^*	0.0013
TT^*	–0.0120
μ^*	0.1393

To attain the standard errors we apply equation 4 to get the covariance matrix

$$\text{cov} = \begin{bmatrix} 0.13931 & 0 & 0 & 0.13102 \\ 0 & 0.13931 & 0 & 0.00134 \\ 0 & 0 & 0.13931 & -0.01205 \\ 0 & 0 & 0 & 0.13931 \end{bmatrix} \cdot \begin{bmatrix} 0.0399 & 0.0004 & -0.0028 & -0.0370 \\ 0.0004 & 0.0001 & -0.0001 & -0.0008 \\ -0.0028 & -0.0001 & 0.0004 & 0.0031 \\ -0.0370 & -0.0008 & 0.0031 & 0.0410 \end{bmatrix} \cdot \begin{bmatrix} 0.13931 & 0 & 0 & 0 \\ 0 & 0.13931 & 0 & 0 \\ 0 & 0 & 0.13931 & 0 \\ 0.13102 & 0.00134 & -0.01205 & 0.13931 \end{bmatrix}$$

$$\text{cov} = \begin{bmatrix} 0.00012749 & -0.00000655 & -0.00000038 & 0.00003028 \\ -0.00000655 & 0.00000172 & -0.00000068 & -0.00000787 \\ -0.00000038 & -0.00000068 & 0.00000331 & -0.00000866 \\ 0.00003028 & -0.00000787 & -0.00000866 & 0.00079570 \end{bmatrix}$$

Finally, the standard errors are obtained by taken the square root of the diagonal elements of the covariance matrix. As expected these results correspond exactly[1] with results in Cranenburgh et al. 2015.

	SE	$t-stat$
FSG^*	0.01123	11.6647
FSO^*	0.00111	1.2075
TT^*	0.00174	−6.9384
μ^*	0.02820	4.9397

[1] In Van Cranenburgh et al. (2015) there is a minor error in Table 5.1 pg 101. The reported t-statistic for the difference from one for the scale parameter equals 30.51, instead of 87.83.

Van Cranenburgh, S., Guevara, C. A. & Chorus, C. G. (2015). New insights on random regret minimization models. *Transportation Research Part A: Policy and Practice*, *74(0)*, 91-109.
Vermunt, J. K. & Magidson, J. (2014). *Upgrade manual for Latent GOLD Choice 5.0*. (Belmont, MA , USA).