Random Regret Minimization

“A short history of nearly everything”

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Random Regret Minimization?

Aims to capture **bounded** rationality in a **tractable** choice model

Bounded rationality

- Reference-dependent preferences
- Loss/Regret aversion
- Semi-compensatory behavior
- Choice set composition effects (non-IIA)

For all aspects, there is compelling empirical evidence + attempts to model

Tractable choice model

- 'Logit-type' choice probabilities (MNL, ML, etc.)
- Identifiable parameters + parsimonity
- No data pre-processing needed

A short history (0): linear RUM (Logit)

Components of a linear-in-parameters RUM-based Logit model

1. Alternatives (car, transit) i, j
2. Attributes (time, cost) x
3. Tastes / weights (for time, cost) – to be estimated β
4. Randomness ε
5. Decision rule (usually not defined explicitly!):

Utility of alternative i: $U_i = V_i + \varepsilon_i = \sum_m \beta_m \cdot x_{im} + \varepsilon_i$

i is chosen iff: $\sum_m \beta_m \cdot x_{im} + \varepsilon_i > \sum_m \beta_m \cdot x_{jm} + \varepsilon_j, \forall j \neq i$

When ε ~ i.i.d. EV Type I across alternatives, choice situations, individuals; with variance $\pi^2/6$:

$$P(i) = P\left(V_i + \varepsilon_i > V_j + \varepsilon_j, \forall j \neq i\right) = \frac{\exp(V_i)}{\sum_{j=1..J} \exp(V_j)} = \frac{\exp(\sum_m \beta_m x_{im})}{\sum_{j=1..J} \exp(\sum_m \beta_m x_{jm})}$$

A short history (1)

PhD-research: traveler response to information

- Risky decision making, learning, information value
- Mathematical model: information reduces "Expected Mistake"
- Moshe Ben-Akiva (personal communication @TRB'05):
  - "why not call it expected regret?"
- Growing obsession with regret literature (Minimax, Regret Theory)
- Harry Timmermans (plenary communication @TRB'06):
  - "why not develop a regret based alternative to linear RUM"
- Position at TU/e (Harry, Theo Arentze): time to do so

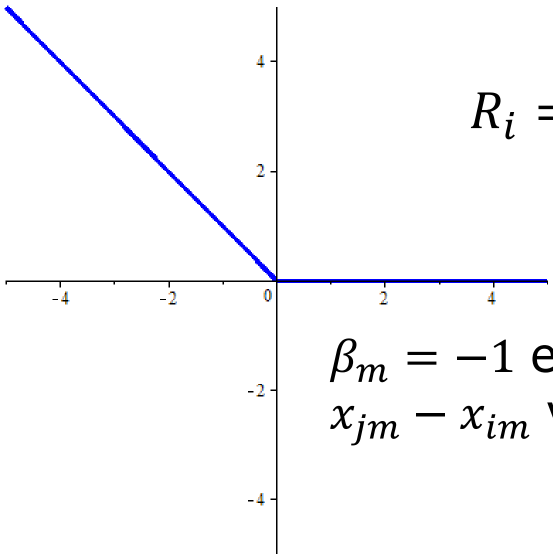
RRM 2008: mathematical notation

$$R_i = \max_{j \neq i} \left\{ \sum_m \max\left\{0, \beta_m \cdot \left(x_{jm} - x_{im}\right)\right\}\right\}$$

- Only regret if > 0; rejoice does not count
- Regret of Alternative i
- Only regret wrt best competing alternative counts
- Repeat and sum over all attributes
- Weigh, according to importance of attribute m (estimable parameter, can be <0)
- Compare i's performance on attribute m, with j's

Chorus, C. G., Arentze, T. A., & Timmermans, H. J. (2008). A random regret-minimization model of travel choice. *Transportation Research Part B: Methodological*, *42*(1), 1-18.

RRM 2008: visual + ‘issues’



$$R_i = \max_{j \neq i}\left\{\sum_m \mathbf{max}\left\{\mathbf{0}, \boldsymbol{\beta}_{\boldsymbol{m}} \cdot \left(\boldsymbol{x}_{\boldsymbol{jm}} - \boldsymbol{x}_{\boldsymbol{im}}\right)\right\}\right\}$$

$\beta_m = -1$ e.g. cost
$x_{jm} - x_{im}$ varied [-5…+5]

Econometric issues:

- kink around zero gives problems with estimation and derivation of elasticities, marginal effects, willingness to pay ($\partial R/\partial\beta$, $\partial R/\partial x$).

Behavioral issues:

- Is ‘rejoice’ really irrelevant?
- Do people only feel regret wrt best non-chosen alternative?

A short history (2)

Considered smooth (Logsum) approximation of max-operator, but no priority.

- Michel Bierlaire (personal communication @ICMC'09):
  - "issues with max are serious; try Logsum"

Also abandoned max-operator across alternatives, for (same) econometric reasons as well as behavioral ones.

Result: smooth, differentiable regret function.

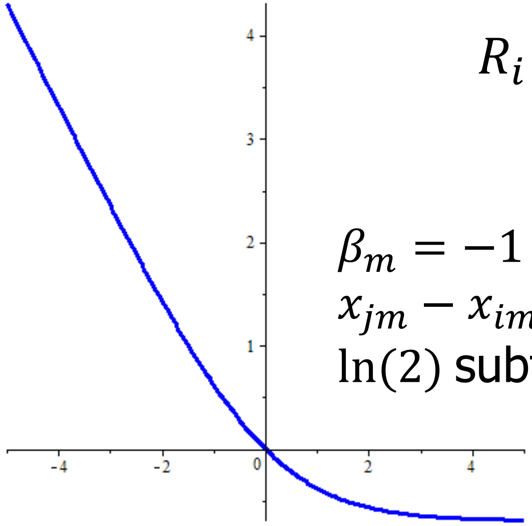
RRM 2010: mathematical notation

$$R_i = \sum_{j \neq i} \sum_m \ln\left(1 + \exp\left[\beta_m \cdot \left(x_{jm} - x_{im}\right)\right]\right)$$

- i: Regret of Alternative i
- $j \neq i$: Repeat and sum over all <u>competing</u> alternatives
- m: Repeat and sum over all attributes
- β_m: Weigh, according to importance of attribute m (estimable parameter, can be <0)
- $x_{jm} - x_{im}$: Compare i's performance on attribute m, with j's

Chorus, C. G. (2010). A new model of random regret minimization. *European Journal of Transport and Infrastructure Research*, 10(2), 181-196.

RRM 2010: visual + interpretation



$$R_i = \sum_{j \neq i} \sum_m \ln\left(1 + \exp\left[\beta_m \cdot \left(x_{jm} - x_{im}\right)\right]\right)$$

$\beta_m = -1$ e.g. cost
$x_{jm} - x_{im}$ varied [-5…+5]
$\ln(2)$ subtracted (inconsequential)

Interpretation:

- Differentiable around zero, so now estimable with Biogeme ;-)
- Rejoice matters
- But less so than regret (in line with Regret Theory, Prospect Theory)
- Regret is experienced wrt all competing alternatives

A short history (3)

This version (and presentation thereof @IATBR'09) sparked series of collaborations, applications. Particular focus on:

- Empirical comparisons with RUM
  - Better model? → In particular contexts? → Different outcomes?
- Decision rule heterogeneity
  - Latent class approaches
- Theoretical properties of RRM
  - Derivation of elasticities, Exploration of compromise effects
- Introducing RRM in other fields
  - Env & Resource Econ., Health Econ., Marketing
- Tackle challenges
  - Sampling of alternatives, Welfare implications/interpretations, …
- Create 'equivalent' of RUM-toolset
  - e.g. RRM based Stochastic User Equilibrium, Route overlap models

RRM: Areas of application

- The usual suspects: (choice for mode, route, departure time, parking lots, etc.)
- Vehicle purchases (regular cars, alternative fuel vehicles)
- Evasive actions on highways (preceding accidents)
- Freight movement (mode to transport goods)
- Evacuation behavior (pedestrians, travelers)
- Policy choices by local politicians
- Voting behavior (Germany)
- Shopping destinations
- Workplace locations
- Nature park visits / tourism destination choices
- Choices for medical treatments of patients
- Lifestyle / dietary choices
- Poaching behavior (Tanzania)
- On-line dating behavior
- …

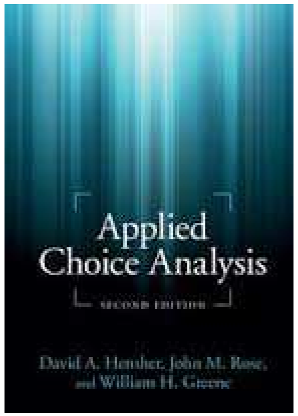
A short history (4)

Gradually, RRM gained a foothold in (part of) the choice modeling community.

Inclusion in software, coverage in Textbooks / Courses certainly helped

Latent GOLD®
Finding gold in your data™

Sawtooth Software



Course in choice modelling and stated choice survey design
London, United Kingdom

A short history (5)

But skepticism remained. And understandably so.

Lack of axiomatic foundation

- Indeed, developed from a behavioral perspective

As a consequence, hard to establish connection with welfare econ.

- Indeed, but some progress made (Thijs Dekker)

Operational difficulties when choice sets are large, or differ in size

- Indeed, but partly solved (e.g. Guevara, Ben-Akiva, Dekker, Hess, van Cranenburgh)

Empirical differences with RUM small or modest (for RRM-2010)

- In many cases: yes. But sometimes not (Hess, Beck), and usually elasticities rather different

Annoying having to choose between RRM and RUM

- Indeed. We need a generalization (or two)

A Generalized RRM Model

$$R_i = \sum_{j \neq i} \sum_m \ln\left(\boldsymbol{\gamma_m} + \exp\left[\beta_m \cdot (x_{jm} - x_{im})\right]\right)$$

Spot the difference: '1' becomes γ_m.

G-RRM exhibits linear RUM-behavior when $\gamma_m = 0$, RRM-2010 behavior when $\gamma_m = 1$. 'In between' behavior when $0 < \gamma_m < 1$.

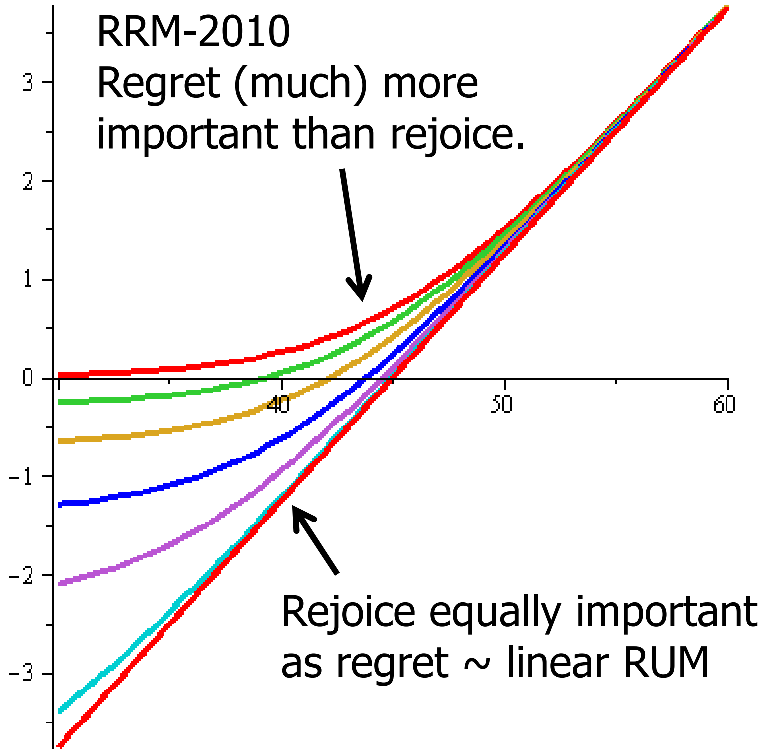
Crucial: γ_m is estimated from data. Regret aversion parameter.

Chorus, C.G., 2014.
Trans. Res. B, 68, 224-238

$\gamma_m = 1$

$0 < \gamma_m < 1$

$\gamma_m = 0$



A short history (6)

But also G-RRM leads to only modest differences with linear RUM
(logical, as it captures only the range between RRM-2010 and RUM)

New generalization based largely on work by Sander van Cranenburgh:

The μRRM model: $R_i = \sum_{j \neq i} \sum_m \boldsymbol{\mu_m} \cdot \ln\left(1 + \exp\left(\frac{\beta_m}{\boldsymbol{\mu_m}}\left[x_{jm} - x_{im}\right]\right)\right)$

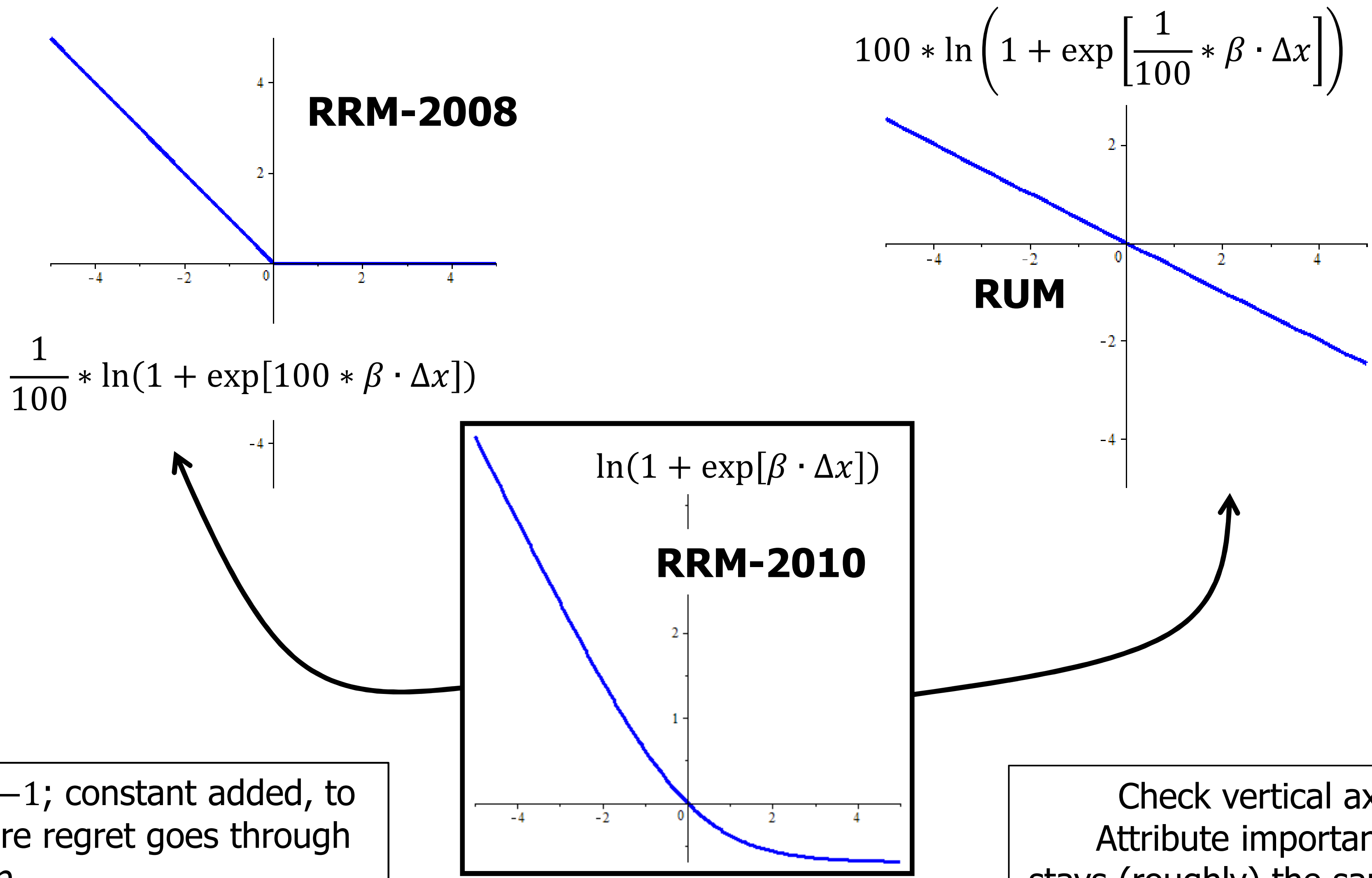
Special cases:

- $\mu \rightarrow 0$: only regret matters; rejoice irrelevant. 'Pure-RRM'. (~RRM-2008)
- $\mu = 1$: RRM-2010
- $\mu \rightarrow +\infty$: regret and rejoice matter equally; linear RUM.

Parameter $\boldsymbol{\mu_m}$ estimable. Range of covered behavior far larger than G-RRM.

van Cranenburgh, S., Guevara, C.A., Chorus, C.G., 2015. New insights on random regret minimization models. *Transportation Research Part A*, 74, 91-109

Range of the μRRM model



$\beta = -1$; constant added, to ensure regret goes through origin.

Check vertical axes: Attribute importance stays (roughly) the same.

Mathematical derivation/interpretation

- $\ln(1 + \exp[\beta \cdot \Delta x])$ originally proposed as a smoothing-function of $\max\{0, \beta \cdot \Delta x\}$
- max-operator caused difficulties with model estimation, derivation of WtP, etc.
- two iid EV Type I-errors added to 0 and $\beta \cdot \Delta x$, respectively; integrated out.
- results in Logsum-formulation (ignoring cnst): $E\left[\max\left(0+\nu_1, \beta \cdot \Delta x + \nu_2\right)\right] = \ln\left(1+\exp[\beta \cdot \Delta x]\right)$
- in doing so, it was implicitly assumed that error-variances (ν) normalized to $\pi^2/6$.
- implicit assumption can be relaxed: variance of implicit errors can be estimated.
- if variance of $\nu = (\pi^2/6) \cdot \mu^2$, $E\left[\max\left(0+\nu_1, \beta \cdot \Delta x + \nu_2\right)\right] = \mu \cdot \ln\left(1+\exp\left[\frac{\beta}{\mu} \cdot \Delta x\right]\right)$
- small (large) variance of implicit errors implies kink (smooth transition) around 0.
- as such, μ determines the 'smoothness', or linearity, of the regret function.

μRRM – shopping location choices

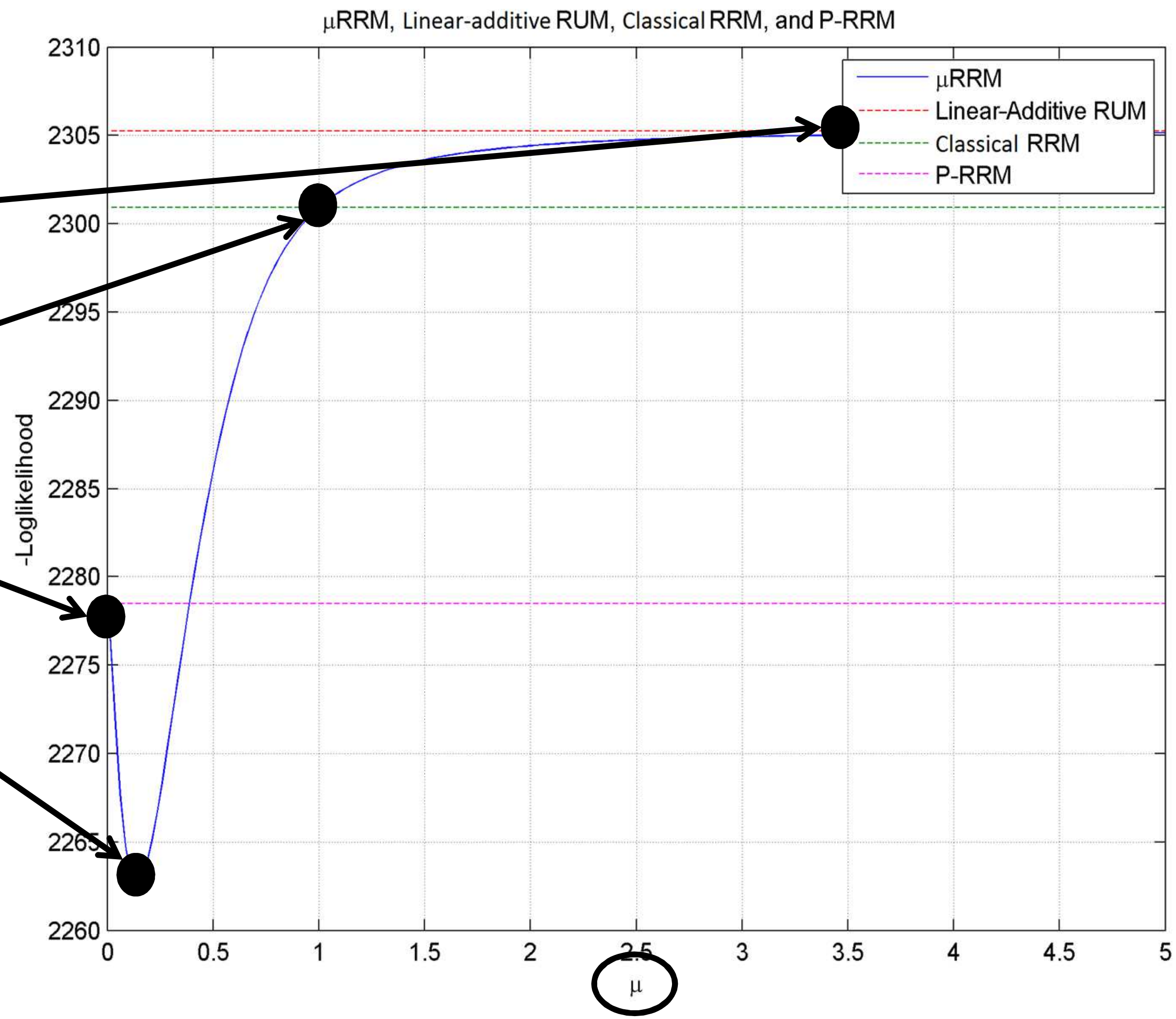
Estimation for diff. values of μ:

Linear RUM fits worst.

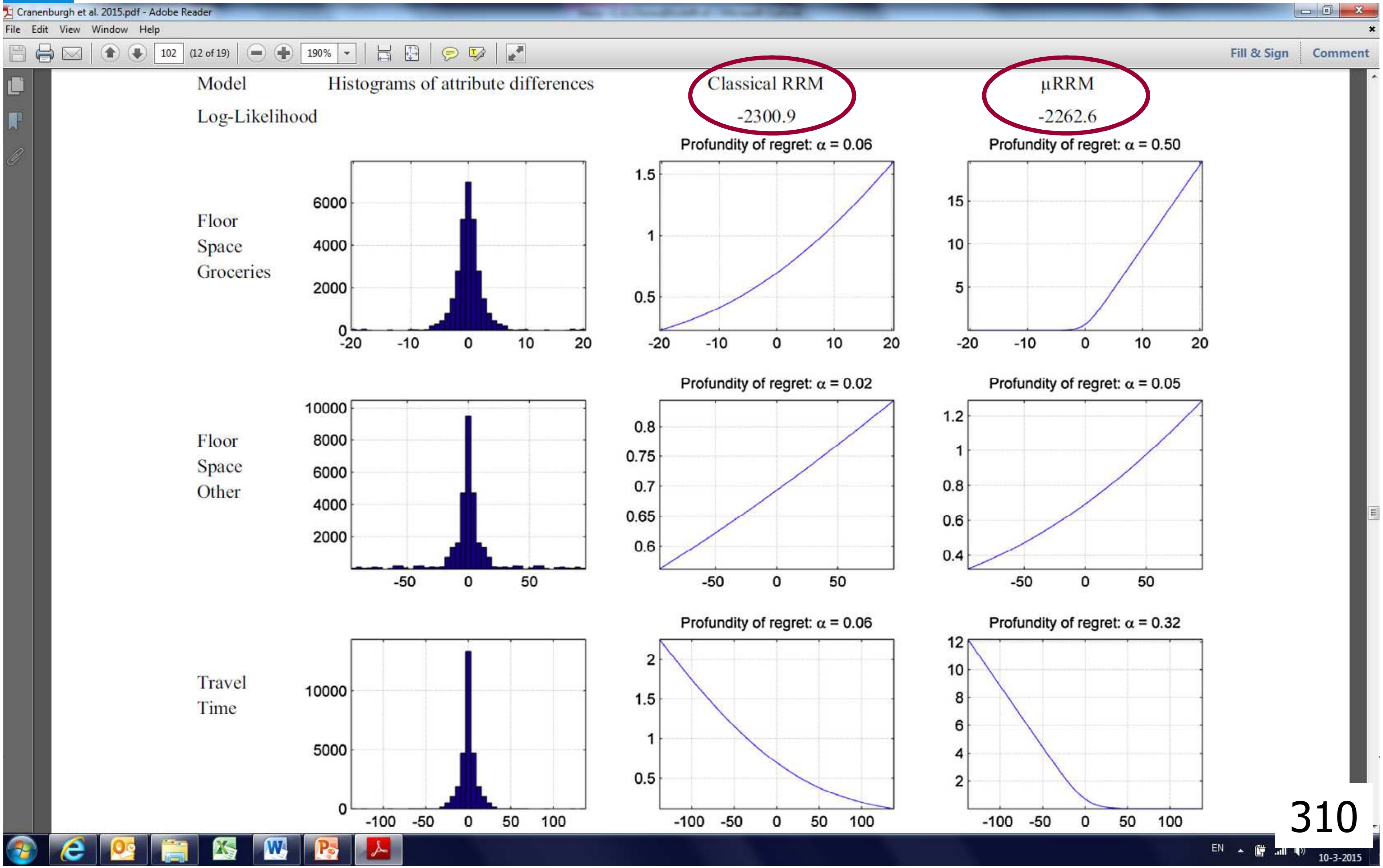
Conventional RRM does somewhat better.

Pure-RRM does a lot Better.

But the best fit is for a model that approaches, yet not equals, Pure-RRM.



Estimating µRRM – shopping location (III)



Estimating μRRM – 10 datasets

Revisited 10 datasets used in previous publications to compare RRM, RUM.

- For datasets where RUM did better than conventional RRM, μRRM reduces to RUM.
- Of the 6 datasets where 2010-RRM did better than RRM:
  - On 2 datasets, μRRM reduces to 2010-RRM
  - On 3 datasets, μRRM achieves values in-between conventional RRM and Pure-RRM
  - On 1 dataset, μRRM reduces to Pure-RRM
- For the last 4 datasets, **model fit improvement found to be very substantial**
  - At the cost of one extra parameter

Conclusions (μRRM):

- Flexible model ('2008-RRM', 2010-RRM, RUM) with behavioral foundation
- Able to pick up extreme levels of regret aversion if present in data
- Much bigger differences in fit, implications, than previous RRM-models
- Useful foundation for future work

Recent / Future work

New model forms (Timmermans), comparison with other models of bounded rationality (Hess)

Connection with welfare economics (Dekker) and optimization (Bierlaire)

Integration in large scale transport models (de Jong, Daly, van Cranenb.)

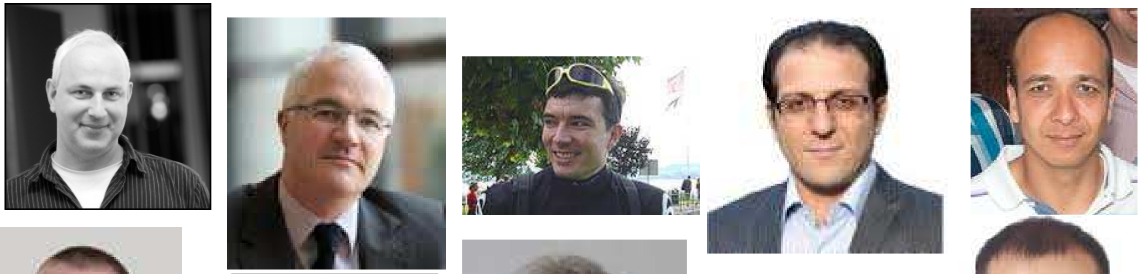
Experimental designs for RRM – this matters!

Study determinants of regret-aversion; can be done easily with μRRM

Beyond NKOTB: model comparisons $\rightarrow$ using (μ)RRM when it does better (acknowledge that linear-additive RUM is also a behavioral assumption!)

You name it… (literally)

Thank you!



Thank you!

Misunderstandings about RRM (1)

I/we do not propose RRM as a better choice model

First, RRM, RUM, RAM, EBA, …, should not be taken too literally

- Not *process* model, but "*as if*": focus on **predicting choice** (probabilities)

Second, behavior (and model performance) is context (data set) specific

- Making RRM a useful **addition** to the choice modeler's toolbox

"it should again be stressed that we do not propose the RRM-approach presented here as superior in any way to RUM-theory. […] The Random Regret-Minimization model presented here is an attempt to provide […] a different perspective. Nothing less, nothing more."

Chorus, C. G., Arentze, T. A., & Timmermans, H. J. (2008). A random regret-minimization model of travel choice. *Transportation Research Part B: Methodological*, *42*(1), 1-18.

Misunderstandings about RRM (2)

RRM is rooted in / application of Regret Theory

RT postulates that regret operates on **utility** ($R_{ij} = Vj - Vi$)
RT focuses on (**single-attribute** and mostly **binary**) **risky** decision contexts
RT attempts to capture **deviations from EUT** (e.g. Allais paradox)

RRM postulates that regret operates on **attributes** ($\ln(1 + \exp[\beta \cdot \Delta x])$)
RRM focuses on **multi-attribute, multinomial** decision contexts
RRM attempts to capture **semi-compensatory behavior** and **choice set composition effects** (e.g. compromise effect)

Own fault: "Our Random Regret Minimization model is rooted in Regret Theory"

Chorus, C. G., Arentze, T. A., & Timmermans, H. J. (2008). A random regret-minimization model of travel choice. *Transportation Research Part B: Methodological*, *42*(1), 1-18.

Some recent estimation results

Choice context	Model fits						
	Linear-additive RUM	RRM2008	RRM2010	G-RRM	γ	μRRM	μ
Parking lots	-406	-368	-405	-405	1	-380	< 0.01
Shopping dest	-2305	-2324	-2301	-2301	1	-2263	0.14
Departure times	-795	-792	-794	-794	1	-792	0.02
Policies	-240	-247	-239	-239	1	-239	1.01
Routes	-1092	-1120	-1094	-1092	0	-1092	4.51
Routes	-1272	-1309	-1278	-1272	0	-1272	> 5
Car types	-4193	-4240	-4194	-4193	0	-4193	> 5
Routes	-4024	-4145	-4038	-4023	0	-4024	> 5
Routes	-2613	-2648	-2605	-2605	1	-2605	0.94
Dating	-3688	-3716	-3648	-3648	1	-3591	0.15