

# Accounting for variation in choice set size in Random Regret Minimization models

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## Abstract

*This paper derives a trick to account for variation in choice set size in Random Regret Minimization (RRM) models. In many choice situations the choice set size varies across choice observations. As in RRM models regret level differences increase with increasing choice set size, not accounting for variation in choice set size results in RRM models to predict relatively deterministic choice behaviour in observations where the choice set is large and relatively random choice behaviour in observations where the choice set is small. Such variation in choice consistency across observations is behaviourally unrealistic and leads to inferior performance of RRM models in the context of data sets with varying choice set sizes. The proposed trick resolves this in an econometrically pragmatic and behaviourally meaningful way by rescaling the regret levels as a function of the choice set size. The trick can be applied in the estimation phase when the choice set size varies across choice observations as well as in the forecasting phase when forecasts are made over choice sets of varying sizes.*

## 1 Introduction

Random Regret Minimization (RRM) models have been proposed as a counterpart of the linear-additive Random Utility Maximization (RUM) model, and are increasingly used to explain and predict a diverse range of choice behaviours (Chorus et al. 2014). The RRM model's recent incorporation in the NLOGIT and Latent GOLD software packages (EconometricSoftware 2012; Vermunt and Magidson 2014), and its inclusion in the second edition of the Applied Choice Analysis textbook (Hensher et al. 2015), can be considered evidence of the growing interest in RRM models among scholars and practitioners. RRM models postulate that regret is experienced when a competing alternative outperforms a considered alternative with regard to one or more attributes. The overall level of regret associated with a considered alternative is postulated to be the sum over all pairwise comparisons between that alternative and all competing alternatives, in terms of all attributes.

In many choice situations the choice set size, i.e. the number of alternatives which are available to the decision-makers, varies across choice observations. In RUM models, variation in choice set size across observations is inconsequential as it does not affect the utilities of alternatives. However, in RRM models variation in choice set size is consequential as it does affect the regret levels of the alternatives. Since the overall regret level equals the *sum* of all pairwise regrets arising from bilateral comparisons with competing alternatives, overall regret levels rise with an increase of the choice set size. Although there is some empirical evidence that larger choice sets potentially lead to more regret from the side of decision-makers (e.g. Schwartz et al. 2002; Sarver 2008), from a discrete choice modelling perspective this phenomenon calls for a modelling intervention from the choice modeller. More specifically, as we will elaborate further below, the rise of regret levels with choice set size predicted by RRM models implies that regret differences between alternatives also tend to grow with the increase of the choice set size. This in turn means that when there is variation in choice set sizes in the data, RRM models predict larger differences in regret levels and therefore more deterministic choice behaviour in observations where the

choice set is relatively large, and smaller differences in regret levels and hence more random choice behaviour in observations where the choice set is relatively small. It goes without saying that such variation in choice consistency is behaviourally unrealistic and leads to inferior model performance of RRM models in the context of data sets with varying choice set sizes (Prato 2014; Mai et al. 2015).

This paper derives a simple trick to account for variation in choice set size in RRM models, which is both econometrically pragmatic and behaviourally meaningful. The trick involves rescaling the regret levels as a function of the choice set size. It can be applied in the estimation phase when the choice set size varies across choice observations as well as in the forecasting phase when forecasts are made over choice sets of varying sizes, or when the choice set used for forecasting is of a different size than the choice set used for estimation. Note that the proposed trick also works in the situation where the number of attributes per alternative (rather than alternatives per choice set) varies across choice observations. For ease of communication, in the remainder of this paper we focus on the case where choice set sizes vary in data used for model estimation. Finally, it should be noted that the proposed trick is particularly suitable for the case where choice sets are relatively large (i.e., consisting of ten or more alternatives) and the composition of the choice set – as opposed to its size – does not vary systematically across observations.<sup>1</sup> In the final section we however also outline a more involved method that can be used to deal with smaller choice sets and systematic variation in choice set composition.

## 2 The effect of variation in choice set size in RRM models

Regret is experienced when a competing alternative  $j$  outperforms a considered alternative  $i$  with regard to attribute  $m$ . The overall regret associated with  $i$  is the sum over all possible pairwise comparisons, see equation 1, where  $n$  denotes the choice observation and  $r_{ijmn}$  the regret experienced in choice observation  $n$  when comparing  $i$  with  $j$  on  $m$ . This so-called attribute level regret can take on various functional forms, depending on the specific type of RRM model under consideration (e.g. Classical RRM,  $\mu$ RRM, or P-RRM, see Van Cranenburgh and Prato (submitted) for an overview of attribute level regret functions).

$RR_{in} = R_{in} + \varepsilon_{in} \text{ where } R_{in} = \sum_{j \neq i} \sum_m r_{ijmn}$	1
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Due to the double summation (equation 1), the overall level of regret  $R_{in}$  increases with the choice set size as well as with the number of attributes, irrespective of the functional form adopted to model attribute regret. This implies that – when the modeller does not correct for the variation in choice set size across choice observations – RRM models impose relatively deterministic choice behaviour in relatively large choice sets, and relatively random choice behaviour in relatively small choice sets. The following example serves to illustrate this point in a simple way.

Suppose that a data set contains choice observations consisting of either 3 or 6 alternatives. For clarity of exposition each alternative consists of just one attribute  $x_j$ . Choice observation 1 consists of a choice from three alternatives {A, B, C} (see Table 1). Choice observation 2 consists of a choice from six alternatives {A, B, C, A, B, C} (see Table 2). Hence, merely for the sake of illustration and without loss of general applicability, we assume that choice set 2 consists

<sup>1</sup> We say that a choice set composition varies in a systematic way across cases, when for example a choice set of size 2 always consists of modes train and bus and a choice set of size 3 always consists of modes train, bus and car. In such a case, the car is systematically absent in the 2 alternatives set, and systematically present in the 3 alternatives set. See the last section for a brief discussion of how to deal with such situations in the context of RRM model estimation.

of exact replicates of the alternatives in choice set 1. Furthermore, we assume that – having estimated the RRM model<sup>2</sup> – the associated taste parameter  $\beta_i$  is found to be equal to one.

Table 1 shows the implied regret levels and predicted choice probabilities in observation 1 (computed using the estimated taste parameter). As can be seen, in observation 1 the RRM model predicts that a decision-maker is  $33/12 = 2.7$  times more likely to choose alternative  $B$  than alternative  $A$ .

	$A$	$B$	$C$
Attribute level $x_i$	0	0.5	1
Regret $R$	1.5	0.5	0
Choice probability $P$	12%	33%	55%

Table 1: Regret levels and choice probabilities for observation 1

Table 2 shows regret levels and predicted choice probabilities for observation 2. Since the choice set in choice observation 2 consists of two exact replications of the choice set of choice observation 1, behavioural intuition suggests that  $P_B / P_A$  in the observation 2 should be equal to  $P_B / P_A$  in observation 1. However, we see that the RRM model in choice observation 2 predicts that a decision-maker is about  $26/4 = 13/2 = 6.5$  times more likely to choose alternative  $B$  over alternative  $A$ .

	$A$	$B$	$C$	$A$	$B$	$C$
Attribute level $x_i$	0	0.5	1	0	0.5	1
Regret $R$	3	1	0	3	1	0
Choice probability $P$	2%	13%	35%	2%	13%	35%

Table 2: Regret levels and choice probabilities for observation 2

Clearly, this kind of differences in predicted randomness across observations is behaviourally unrealistic and translates into inferior model fit for RRM models when choice set sizes vary across observations in the data used for model estimation.

### 3 A trick to account for variation in choice set size in RRM models

#### 3.1 The trick

Equation 2 shows a simple and effective trick to account for variation in the choice set size when estimating RRM models. The overall regret level is corrected using a factor  $\Gamma / J_n$ , where  $J_n$  denotes the size of the choice set presented to the decision-maker in observation  $n$ , and  $\Gamma$  denotes a constant. For the stylized example presented above (and in Appendix A), this correction factor yields constant ratios of the choice probabilities of any two alternatives, regardless of the size of the choice set (i.e., the number of replications)<sup>3</sup>, see Appendix A for a formal proof. While it is

<sup>2</sup> In this illustration, without loss of general applicability, we use the P-RRM model (Van Cranenburgh et al. 2015) to compute regret levels.

<sup>3</sup> Note that also with the correction factor, the RRM model predicts a violation of the IIA property. This is by design, as the RRM model aims to capture choice set composition effects such as the compromise effect. The numerical example

clear that the stylized situation presented above is unlikely to occur in real life, it nonetheless provides a good indication that the correction factor in equation 2 should also work well on real data. This suggestion is indeed confirmed on a series of empirical analysis based on real as well as simulated data (not reported here for reasons of space limitations).

$\tilde{R}_{ni} = \frac{\Gamma}{J_n} R_{ni}$	2
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It is important at this point to note that while for some types of RRM models the choice of  $\Gamma$  is consequential for the behaviour imposed by the model, for other types it is not. More specifically, for the  $\mu$ RRM and P-RRM model (Van Cranenburgh et al. 2015) the choice of  $\Gamma$  is not consequential. In the  $\mu$ RRM model the scale parameter  $\mu$  is estimated, as opposed to being implicitly fixed to 1 as in the Classical RRM model (Chorus 2010). Since  $\Gamma$  is perfectly confounded with  $\mu$ , setting  $\Gamma$  to a large value will merely result in estimating a small scale parameter  $\mu$ , and vice versa. Therefore, in a  $\mu$ RRM model the imposed behaviour is not affected by the choice of  $\Gamma$ , implying that  $\Gamma$  can freely be chosen by the choice modeller. Likewise, the choice of  $\Gamma$  is inconsequential for the behaviour imposed by the P-RRM model (which is a special case of the  $\mu$ RRM model). Since the attribute level regret function of the P-RRM model is scale-invariant, it always imposes the same degree of regret minimizing behaviour, irrespective of the choice of  $\Gamma$ .

In contrast, the size of  $\Gamma$  is consequential for the Classical RRM model (Chorus 2010) and for the G-RRM model (Chorus 2014). Since the attribute level regret functions of these two RRM models are not scale-invariant, a different choice of  $\Gamma$  leads to different degrees of regret minimizing behaviour imposed by the model, and hence a different empirical performance. More specifically, a large (small) value of  $\Gamma$  causes parameters to become small (large), leading to a model which imposes a mild (strong) degree of regret minimizing behaviour (see Van Cranenburgh et al. (2015) for a discussion of the relation between parameter sizes and the resulting degree of regret minimizing behaviour). More generally speaking, the fact that  $\Gamma$  cannot freely be chosen without affecting the choice behaviour imposed by the model, is very much related to the observation made recently that the scale underlying the Classical (and the Generalized) RRM model, which is implicitly fixed to 1, can and should be estimated (Van Cranenburgh et al. 2015). The resulting  $\mu$ RRM model provides a more flexible account of choice behaviour, and – as we have highlighted directly above – it features the related additional advantage of making the choice for a particular of  $\Gamma$  inconsequential, as opposed to being both arbitrary and consequential for the Classical RRM and Generalized RRM models.

Finally, it is important to stress once more that the proposed trick is simple, yet somewhat coarse in the sense that the proposed correction factor is only a function of the choice set *size* and does not take into account the *composition* of the choice set. As a result, for the trick to perform well the choice set sizes need to be relatively large<sup>4</sup> and the exact composition of the choice sets should not be systematic across observations.

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used in Section 2 involves exact choice set replications to avoid confounding between this wanted violation of IIA, and the unwanted violation caused by differences in implied randomness in behaviour.

<sup>4</sup> Based on our experience, the trick seems to work well empirically for choice sets consisting of 10 or more alternatives, and somewhat less so for smaller choice set sizes.

### 3.2 Related approaches in the RRM-literature

In recent papers, two related factors have been proposed which in a mathematical sense are special cases of the correction factor proposed in equation 2. More specifically, when  $\Gamma$  is set to one, our correction factor is equal to the factor presented by Mai et al. (2015) in the context of a recursive logit route choice model based on the G-RRM model. In their model – which they coin the Average RRM (ARRM) model – regret levels are normalized by dividing each alternative’s regret by the choice set size. As we discuss in the previous sub-section, setting  $\Gamma$  to the arbitrarily chosen value of 1 implies in the context of the G-RRM used by Mai et al. (2015) a highly specific – yet arbitrarily chosen – degree of regret minimizing behaviour.

In the conceptually different but mathematically related context of choice set sampling, Guevara et al. (in press) proposed an expansion factor  $w = J/\tilde{J}$  to rescale regret, where  $J$  denotes the size of the universal choice set, and  $\tilde{J}$  denotes the size of the sampled choice set which is used in the estimator. They show that when using this factor, consistent parameters are obtained from the sampled choice set. Note that when  $\Gamma$  is set equal to the size of the largest choice set present in the data, our correction factor for dealing with varying choice set sizes within a dataset resembles the expansion factor presented by Guevara et al. (in press) in the rather different context of choice set sampling<sup>5</sup>. But note also here, that setting  $\Gamma$  to equal the choice set size of the largest choice set available in the data leads to a very specific degree of regret minimizing behaviour when either the Classical or the G-RRM models are being used.

## 4 Conclusion and discussion

This paper has proposed a correction factor to account for variation in choice set size across choice observations in the context of RRM models. The method can be used in the estimation phase when the choice set size varies across observations as well as in the forecasting phase when forecasts need to be made over choice sets of varying sizes (or when the size of the choice set used for forecasting differs from the size of the choice set used for estimation). Furthermore, we have shown that the proposed correction factor is generic in that it nests – in a conceptual sense – two related factors that have recently been proposed in the broader RRM literature.

We have outlined that the proposed correction factor can be expected to work well when the choice sets are relatively large and the choice set compositions are unsystematic. When the choice sets are relatively small, or the choice set compositions are systematic, a different method is needed. One promising possibility to deal with such a situation is to estimate *choice set size specific correction parameters* – rather than using a generic correction factor like we propose in this paper. This method is akin to the way in which the scale of choice models needs to be adjusted when different data sets are pooled (see e.g. Ben-Akiva and Morikawa 1990; Ben-Akiva et al. 1994). Further research is needed to further explore this method in the context of data sets where choice sets are relatively small, or the choice set compositions are systematic. However, first results obtained in the context of the Swiss Metro dataset (Antonini et al. 2007) appear promising (see Appendix B).

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<sup>5</sup> But note that in our situation, smaller choice sets may include alternatives that are not present in the larger choice set, so that there is no actual sampling process.

## Appendix A

Suppose that a data set consists of observations having choice set  $C_1$  and choice observations having choice set  $C_2$ . Choice set 1  $C_1 = \{a, b, \dots, z\}$  consists of  $Z$  alternatives, and choice set 2  $C_2 = \{C_1, C_1, C_1, \dots, C_1\}$  consists of  $L$  exact replications of set  $C_1$ .

**Theorem:** To account for variation in the choice set size in the  $\mu$ RRM model and its special cases (the P-RRM model and the Classical RRM model) such that the ratio of choice probabilities of any two alternatives  $a$  and  $b$  is constant, regardless of the choice set size (i.e. regardless of the number of replications of  $C_1$ ), regret levels need to be scaled with a factor  $\Gamma/J_n$ , where  $J_n$  denotes the choice set size, and  $\Gamma$  is a constant.

### Proof:

The ratios of the choice probabilities of alternatives  $a$  and  $b$  in choice set  $C_1$  and choice set  $C_2$  are given in equation A.1:

$\frac{P(a   C_1)}{P(b   C_1)} = \frac{e^{-\mu \sum_{\substack{j \neq a \\ j \in C_1}} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right)}}{e^{-\mu \sum_{\substack{j \neq b \\ j \in C_1}} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right)}}$	$\frac{P(a   C_2)}{P(b   C_2)} = \frac{e^{-\mu \sum_{\substack{j \neq a \\ j \in C_2}} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right)}}{e^{-\mu \sum_{\substack{j \neq b \\ j \in C_2}} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right)}}$	A.1
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Acknowledging that  $C_1$  is a proper subset of  $C_2$  we can rewrite the ratio of the choice probabilities of alternative  $a$  and  $b$  in choice set  $C_2$  using  $C_1$  (equation A.2).

$\frac{P(a   C_2)}{P(b   C_2)} = \frac{e^{-\mu \left[ L \cdot \sum_{\substack{j \neq a \\ j \in C_1}} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right) \right] - (L-1) \cdot \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{am} - x_{am}] \right) \right)}}{e^{-\mu \left[ L \cdot \sum_{\substack{j \neq b \\ j \in C_1}} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right) \right] - (L-1) \cdot \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{bm} - x_{bm}] \right) \right)}}$	A.2
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Noting that  $\ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{am} - x_{am}] \right) \right) = \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{bm} - x_{bm}] \right) \right) = \ln(2)$ , and that constants are irrelevant in discrete choice models, equation A.2 reduces to equation A.3.

$\frac{P(a   C_2)}{P(b   C_2)} = \frac{e^{-\mu \left[ L \cdot \sum_{\substack{j \neq a \\ j \in C_1}} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right) \right]}}{e^{-\mu \left[ L \cdot \sum_{\substack{j \neq b \\ j \in C_1}} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right) \right]}}$	A.3
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From equation A.3 it is easily seen that to ensure that  $\frac{P(a|C_1)}{P(b|C_1)} = \frac{P(a|C_2)}{P(b|C_2)}$  holds, regrets

levels in A.3 need to be divided by  $L$ . Without this division, the ratio becomes much larger for set  $C_2$ , and more so when  $L$  increases, implying that the choice probability for the least (most) attractive alternative approaches 0 (1) as  $L$  becomes very large.

In practical applications, choice sets are almost never exact replications of one another. Therefore, instead of using the number of replications  $L$  of a given choice set to correct the regret levels, it is more practical to write the correction factor as a function of the choice set size. The choice set size of  $C_1$  and  $C_2$  is respectively  $Z$  and  $L \times Z$ . Equation A.4 and A.5 show that when a correction factor  $\Gamma/J_n$  is used the ratios of the choice probabilities of  $a$  over  $b$  are independent of the choice set size, where  $\Gamma$  denotes a constant.

$\frac{P(a C_1)}{P(b C_1)} = \frac{e^{-\mu \frac{\Gamma}{J_n} \sum_{j \in C_1} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right)}}{e^{-\mu \frac{\Gamma}{J_n} \sum_{j \in C_1} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right)}} = \frac{e^{-\mu \frac{\Gamma}{Z} \sum_{j \in C_1} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right)}}{e^{-\mu \frac{\Gamma}{Z} \sum_{j \in C_1} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right)}}$	A.4
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$\frac{P(a C_2)}{P(b C_2)} = \frac{e^{-\mu \frac{\Gamma}{J_n} \left[ L \cdot \sum_{j \in C_1} \sum_{j \neq a} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right) \right]}}{e^{-\mu \frac{\Gamma}{J_n} \left[ L \cdot \sum_{j \in C_1} \sum_{j \neq b} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right) \right]}} = \frac{e^{-\mu \frac{\Gamma}{Z \cdot L} \left[ L \cdot \sum_{j \in C_1} \sum_{j \neq a} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right) \right]}}{e^{-\mu \frac{\Gamma}{Z \cdot L} \left[ L \cdot \sum_{j \in C_1} \sum_{j \neq b} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right) \right]}}$ $= \frac{e^{-\mu \frac{\Gamma}{Z} \sum_{j \in C_1} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{am}] \right) \right)}}{e^{-\mu \frac{\Gamma}{Z} \sum_{j \in C_1} \sum_m \ln \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{bm}] \right) \right)}}$	A.5
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Finally, it is important to mention once again that in principle the size of  $\Gamma$  is free to choose by the choice modeller (e.g. set to one). However, it is important to acknowledge that  $\Gamma$  is confounded with the scale parameter  $\mu$ . As a result of that, in the some RRM models (more specifically, in the Classical RRM and the G-RRM model) the choice of  $\Gamma$  is consequential for the imposed behaviour.

Q.E.D.

## Appendix B

This appendix explores a method that can be used to account for variation in choice set size in the context of small choice sets, or when the composition of the choice set varies systematically across observations. This method involves estimating *choice set size specific correction parameters*, denoted  $\Lambda$ , and is akin to the way in which the scale of choice models needs to be adjusted when different data sets are pooled (see e.g. Ben-Akiva and Morikawa 1990; Ben-Akiva et al. 1994).

### Data

To explore this method we use Stated Preference mode choice data collected in Swiss in 1998. In this experiment respondents were presented choice sets consisting of either two or three alternatives: a conventional train alternative, a Swiss metro alternative (a mag-lev underground system), and a car alternative available only to car owners. Out of the total 6768 choice observations, 1131 have choice set size 2 and the remaining 5607 have choice set size 3. Alternatives are defined in terms of two attributes: travel cost and travel time. See Antonini et al. (2007) for a more detailed discussion on the data set. Finally, note that the choice set composition in these data is highly systematic. Therefore, the method proposed in the main text of this paper can be expected to perform less well for these data.

### Model specification

We estimated three models: (1) a linear-additive RUM model, (2) a  $\mu$ RRM model without correction factors, and (3) a  $\mu$ RRM model with choice set size specific correction factors. In order to identify the latter model, one of the choice set size specific correction factors needs to be fixed. Therefore, we fix the correction factor associated with choice set of size 2,  $\Lambda_{J=2}$ , to one, and estimate the correction factor associated with choice set of size 3:  $\Lambda_{J=3}$  (jointly with the models' taste parameters).

### Results

Table B.1 shows the estimation results. Three key observations can be made. Firstly, in terms of model fit, the results show that both  $\mu$ RRM models substantially outperform the RUM model. Secondly, looking more closely at the  $\mu$ RRM models, we see a very significant increase in log-likelihood for the  $\mu$ RRM model with the choice set size specific correction factors as compared to the model without correction factors. Thirdly, the estimated choice set specific correction factor  $\Lambda_{J=3}$  is considerably larger than one:  $\Lambda_{J=3} = 3.60$ . This indicates that the correction factor discussed in the main text of the paper – which would impose a correction factor of 2/3 in this case – would result in inferior empirical performance.

Finally, it is important to mention that by estimating choice set size specific correction parameters also heteroskedasticity across choice sets of different size may be picked up by the model. To investigate the extent to which such heteroskedasticity may have been captured, we analyse whether the behaviour imposed by the model (i.e., the degree of regret minimizing behaviour) is constant across the choice sets of size 2 and 3. Table B.2. shows the profundity of regret (see Van Cranenburgh et al. 2015) associated with travel cost and travel time. As can be seen, the imposed behaviour is not the same across the choice set sizes: the profundity of regret associated with travel time is substantially higher in choice sets of size 3 than in choice sets of size 2. This suggests that the model has captured some heteroskedasticity across choice sets of different size by accounting for variation in choice set size using choice set size specific correction parameters on these data.

**Swiss Metro data**

MODEL	RUM MNL			$\mu$ RRM-MNL					
				Without correction factor			With choice set size specific correction factor		
Number of observations				6768			6768		
Null Log-likelihood	-6964.7			-6964.7			-6964.7		
Final Log-likelihood	-5331.3			-5264.9			-5145.8		
$\rho^2$	0.235			0.244			0.261		
	<i>Est</i>	<i>SE</i>	<i>t-stat</i>	<i>Est</i>	<i>SE</i>	<i>t-stat</i>	<i>Est</i>	<i>SE</i>	<i>t-stat</i>
<i>Alternative Specific Constants</i>									
Car	0.00 --fixed--			0.00 --fixed--			0.00 --fixed--		
SM	0.16	0.043	3.58	-0.06	0.030	-1.92	-0.21	0.07	-3.08
Train	-0.55	0.046	-11.85	0.29	0.088	3.30	0.75	0.20	3.77
<i>Taste parameters</i>									
$\beta_{\text{COST}}$	-1.08	0.052	-20.91	-0.76	0.036	-21.08	-0.22	0.03	-7.14
$\beta_{\text{TIME}}$	-1.28	0.057	-22.46	-0.99	0.042	-23.53	-0.25	0.04	-6.89
<i>Correction and scale parameters</i>									
$\Lambda_{J=2}$							1.00 --fixed--		
$\Lambda_{J=3}$							3.60	0.48	7.51
$\mu$				1.87	0.548	3.41	0.34	0.10	3.39

Table B.1: Estimation result for choice set size specific correction factor

	Choice set size	
	J = 2	J = 3
$\alpha_{\text{COST}}$	0.24	0.23
$\alpha_{\text{TIME}}$	0.06	0.12

Table B.2 Profundity of regret

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