Accounting for variation in choice set size in Random Regret Minimization models

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Abstract
This paper derives a correction factor to account for variation in choice set size in Random Regret Minimization (RRM) models. In many choice situations the choice set size varies across choice observations. As in RRM models regret level differences increase with increasing choice set size, not accounting for variation in choice set size results in RRM models to predict relatively deterministic choice behaviour in observations where the choice set is large and relatively random choice behaviour in observations where the choice set is small. Such variation in choice consistency across observations is behaviourally unrealistic and leads to inferior performance of RRM models in the context of data sets with varying choice set sizes. The proposed correction factor resolves this in an econometrically pragmatic and behaviourally meaningful way by rescaling the regret levels as a function of the choice set size. The correction factor can be applied in the estimation phase when the choice set size varies across choice observations as well as in the forecasting phase when forecasts are made over choice sets of varying sizes.

1 Introduction

Random Regret Minimization (RRM) models have been proposed as a counterpart of the linear-additive Random Utility Maximization (RUM) model, and are increasingly used to explain and predict a diverse range of choice behaviours (Chorus et al. 2014). The RRM model's recent incorporation in the NLOGIT and Latent GOLD software packages (EconometricSoftware 2012; Vermunt and Magidson 2014), and its inclusion in the second edition of the Applied Choice Analysis textbook (Hensher et al. 2015), can be considered evidence of the growing interest in RRM models among scholars and practitioners. RRM models postulate that regret is experienced when a competing alternative outperforms a considered alternative with regard to one or more attributes. The overall level of regret associated with a considered alternative is postulated to be the sum over all pairwise comparisons between that alternative and all competing alternatives, in terms of all attributes.

In many choice situations the choice set size, i.e. the number of alternatives which are available to the decision-makers, varies across choice observations. In RRM models such variation in choice set size is consequential for the model predictions[1]. Since the overall regret level of an alternative equals the *sum* of all pairwise regrets arising from bilateral comparisons with competing alternatives, overall regret levels rise with an increase of the choice set size. Although there is some empirical evidence that larger choice sets potentially lead to more regret from the decision-makers' perspective (e.g. Schwartz et al. 2002; Sarver 2008), from a discrete choice modelling perspective this phenomenon calls for a modelling intervention from the choice modeller. More specifically, as we will elaborate further below, the rise of regret levels with choice set size predicted by RRM models implies that regret differences between alternatives also tend to grow with the increase of the choice set size. This in turn means that when there is variation in choice

[1] Note that also in RUM models correction factors may be needed, for instance, to deal with differences in choice set composition between estimation and forecasting (see e.g. Daly 1982).

set sizes in the data, RRM models predict larger differences in regret levels and therefore more deterministic choice behaviour in observations where the choice set is relatively large, and smaller differences in regret levels and hence more random choice behaviour in observations where the choice set is relatively small. It goes without saying that such variation in choice consistency is behaviourally unrealistic and leads to inferior model performance of RRM models in the context of data sets with varying choice set sizes (Prato 2014; Mai et al. 2015).

This paper derives a correction factor to account for variation in choice set size in RRM models, which is both econometrically pragmatic and behaviourally meaningful. The correction factor involves rescaling the regret levels as a function of the choice set size. It can be applied in the estimation phase when the choice set size varies across choice observations as well as in the forecasting phase when forecasts are made over choice sets of varying sizes, or when the choice set used for forecasting is of a different size than the choice set used for estimation. Note that the proposed correction factor also works in the situation where the number of attributes per alternative (rather than alternatives per choice set) varies across choice observations. For ease of communication, in the remainder of this paper we focus on the case where choice set sizes vary in data used for model estimation. Finally, it should be noted that the proposed correction factor is particularly suitable for the case where choice sets are relatively large (i.e., consisting of 10 or more alternatives) and the composition of the choice set – as opposed to its size – does not vary systematically across observations.[2] In the final section we however also outline a more involved method that can be used to deal with smaller choice sets and systematic variation in choice set composition.

2 The effect of variation in choice set size in RRM models

Regret is experienced when a competing alternative *j* outperforms a considered alternative *i* with regard to attribute *m*. The overall regret associated with *i* is the sum over all possible pairwise comparisons, see equation 1, where *n* denotes the choice observation and r_{ijmn} the regret experienced in choice observation *n* when comparing *i* with *j* on *m*. This so-called attribute level regret can take on various functional forms, depending on the specific type of RRM model under consideration (e.g. classical[3] RRM, μRRM, or P-RRM, see Van Cranenburgh and Prato (submitted) for an overview of attribute level regret functions).

$$RR_{in} = R_{in} + \varepsilon_{in} \text{ where } R_{in} = \sum_{j \neq i} \sum_{m} r_{ijmn} \quad (1)$$

Due to the double summation (equation 1), the overall level of regret R_{in} increases with the choice set size as well as with the number of attributes, irrespective of the functional form adopted to model attribute regret. This implies that when the modeller does not correct for the variation in choice set size across choice observations RRM models impose relatively deterministic choice behaviour in relatively large choice sets, and relatively random choice behaviour in relatively small choice sets. The following example serves to illustrate this point in a simple way.

Suppose that a data set contains choice observations consisting of either 3 or 6 alternatives. For

[2] We say that a choice set composition varies in a systematic way across cases, when for example a choice set of size 2 always consists of modes train and bus and a choice set of size 3 always consists of modes train, bus and car. In such a case, the car is systematically absent in the 2 alternatives set, and systematically present in the 3 alternatives set. See the last section for a brief discussion of how to deal with such situations in the context of RRM model estimation.

[3] The RRM model proposed in Chorus (2010) is referred to as 'classical' to distinquish between this model and other types of RRM-models that have more recently been proposed.

clarity of exposition each alternative consists of just one attribute x_1. Choice observation 1 consists of a choice from three alternatives {A1, B1, C1} (see Table 1). Choice observation 2 consists of a choice from six alternatives {A1, B1, C1, A2, B2, C2} (see Table 2), where A2, B2, and C2 are exact replicates of respectively A1, B1, and C1. Hence, merely for the sake of illustration and without loss of general applicability, we assume that choice set 2 consists of exact replicates of the alternatives in choice set 1. Furthermore, we assume that – having estimated the RRM model[4] – the associated taste parameter β_1 is found to be equal to one.

Table 1 shows the implied regret levels and predicted choice probabilities in observation 1 (computed by using the estimated taste parameter). As can be seen, in observation 1 the RRM model predicts that a decision-maker is 33/12 = 2.7 times more likely to choose alternative *B1* than alternative *A1*.

	A1	*B1*	*C1*
Attribute level x_1	0	0.5	1
Regret *R*	1.5	0.5	0
Choice probability *P*	12%	33%	55%

Table 1: Regret levels and choice probabilities for observation 1

Table 2 shows regret levels and predicted choice probabilities for observation 2. Since the choice set in choice observation 2 consists of two exact replications of the choice set of choice observation 1, behavioural intuition suggests that P_{B1}/P_{A1} in observation 2 should be equal to $(P_{B1}+P_{B2})/(P_{A1}+P_{A2})$ in observation 1. However, we see that the RRM model in choice observation 2 predicts that a decision-maker is about 26/4 = 13/2 = 6.5 times more likely to choose a *B* alternative over an *A* alternative.

	A1	*B1*	*C1*	*A2*	*B2*	*C2*
Attribute level x_1	0	0.5	1	0	0.5	1
Regret *R*	3	1	0	3	1	0
Choice probability *P*	2%	13%	35%	2%	13%	35%

Table 2: Regret levels and choice probabilities for observation 2

Clearly, this kind of differences in predicted randomness across observations is behaviourally unrealistic and translates into inferior model fit for RRM models when choice set sizes vary across observations in the data used for model estimation.

3 A correction factor to account for variation in choice set size in RRM models

3.1 The correction factor

Equation 2 shows a simple and effective correction factor to account for variation in the choice set size when estimating RRM models. The overall regret level is corrected using a factor Γ / J_n,

[4] In this illustration, without loss of general applicability, we use the P-RRM model (Van Cranenburgh et al. 2015) to compute regret levels.

where J_n denotes the size of the choice set presented to the decision-maker in observation n, and Γ denotes a constant. For the stylized example presented above (and in Appendix A), this correction factor yields constant ratios of the choice probabilities of any two alternatives, regardless of the size of the choice set (i.e., the number of replications)[5], see Appendix A for a formal proof. While it is clear that the stylized situation presented above is unlikely to occur in real life, it provides support for the notion that this correction factor will also work *reasonably* well in the context of more realistic choice situations that do not consist of exact replications of the choice sets. This suggestion is indeed confirmed on a series of empirical analyses based on real as well as simulated data (not reported here for reasons of space limitations).

$$\tilde{R}_{ni} = \frac{\Gamma}{J_n} R_{ni} \quad (2)$$

It is important at this point to note that while for some types of RRM models the choice of Γ is consequential for the behaviour imposed by the model, for other types it is not. More specifically, for the µRRM and P-RRM model (Van Cranenburgh et al. 2015) the choice of Γ is inconsequential. In the µRRM model the scale parameter μ is estimated, as opposed to being implicitly fixed to 1 as in the classical RRM model (Chorus 2010). Since Γ is perfectly confounded with μ, setting Γ to a large value will merely result in estimating a small scale parameter μ, and vice versa. Therefore, in a µRRM model the imposed behaviour is not affected by the choice of Γ, implying that Γ can freely be chosen by the choice modeller. Likewise, the choice of Γ is inconsequential for the behaviour imposed by the P-RRM model (which is a special case of the µRRM model). Since the attribute level regret function of the P-RRM model is scale-invariant, it always imposes the same degree of regret minimizing behaviour, irrespective of the choice of Γ.

In contrast, the size of Γ is consequential for the classical RRM model (Chorus 2010) and for the G-RRM model (Chorus 2014). Since the attribute level regret functions of these two RRM models are not scale-invariant, a different choice of Γ leads to different degrees of regret minimizing behaviour imposed by the model, and hence a different empirical performance. More specifically, a large (small) value of Γ causes parameters to become small (large), leading to a model which imposes a mild (strong) degree of regret minimizing behaviour (see Van Cranenburgh et al. (2015) for a discussion of the relation between parameter sizes and the resulting degree of regret minimizing behaviour). More generally speaking, the fact that Γ cannot freely be chosen without affecting the choice behaviour imposed by the model, is very much related to the observation made recently that the scale underlying the classical (and the Generalized) RRM model, which is implicitly fixed to 1, can and should be estimated (Van Cranenburgh et al. 2015). The resulting µRRM model provides a more flexible account of choice behaviour, and – as we have highlighted directly above – it features the related additional advantage of making the choice for a particular of Γ inconsequential, as opposed to being both arbitrary and consequential for the classical RRM and Generalized RRM models.

Finally, it is important to stress once more that the proposed correction factor is simple, yet somewhat coarse in the sense that it is only a function of the choice set *size* and does not take into account the *composition* of the choice set. As a result, for the correction factor to perform well the choice set sizes need to be relatively large and the exact composition of the choice sets

[5] Note that also with the correction factor, the RRM model predicts a violation of the IIA property. This is by design, as the RRM model aims to capture choice set composition effects such as the compromise effect. The numerical example used in Section 2 involves exact choice set replications to avoid confounding between this wanted violation of IIA, and the unwanted violation caused by differences in implied randomness in behaviour.

should not be systematic across observations. The correction factor is found to perform better empirically in the context of large choice sets (i.e. choice sets of 10 or more alternatives) than in the context of small choice sets. This is due to the fact that in relatively large choice sets the effect of specific choice set constellations (in terms of the combinations of attribute levels of alternatives) on the overall regret levels is averaged out. This implies that the impact of the specific choice set composition on the average choice-set-size-corrected overall regret levels diminishes with increasing choice set size.

3.2 Related approaches in the RRM-literature

Two related factors which in a mathematical sense are special cases of the correction factor proposed in equation 2 have been proposed in recent literature. More specifically, when Γ is set to one, our correction factor is equal to the factor presented by Mai et al. (2015) in the context of a recursive logit route choice model based on the G-RRM model. In that model – which they coined the Average RRM (ARRM) model – regret levels are normalized by dividing each alternative's regret by the choice set size. In the conceptually different but mathematically related context of choice set sampling, Guevara et al. (in press) proposed in the context of the classical RRM model an expansion factor $w = J / \tilde{J}$ to rescale regret, where J denotes the size of the universal choice set, and $\tilde{J}$ denotes the size of the sampled choice set which is used in the estimator. They showed that when using this factor, consistent parameters are obtained from the sampled choice set in the context of estimation of a classical RRM model. Note that when Γ is set equal to the size of the largest choice set present in the data, our correction factor for dealing with varying choice set sizes within a dataset resembles the expansion factor presented by Guevara et al. (in press) in the rather different context of choice set sampling[6].

Hence, to account for choice set size effects in RRM model both studies have made a different (implicit) choice on the size of Γ. Yet, as we have pointed out in the previous sub-section, since these studies are based on the G-RRM model and on the classical RRM the size of Γ is not arbitrary to choose. This implies that in the context of those models using a different correction factor would have resulted in substantially different degrees of regret minimizing behaviour.

4 Conclusion and discussion

This paper has proposed a correction factor to account for variation in choice set size across choice observations in the context of RRM models. The method can be used in the estimation phase when the choice set size varies across observations as well as in the forecasting phase when forecasts need to be made over choice sets of varying sizes (or when the size of the choice set used for forecasting differs from the size of the choice set used for estimation). Furthermore, we have shown that the proposed correction factor is generic in that it nests – in a conceptual sense – two related factors that have recently been proposed in the broader RRM literature.

We have outlined that the proposed correction factor can be expected to work well when the choice sets are relatively large and the choice set compositions are unsystematic. When the choice sets are relatively small, or the choice set compositions are systematic, a different method is needed. One promising possibility to deal with such as situation is to estimate *choice set size specific correction parameters* – rather than using a generic correction factor like we propose in this paper. This method is akin to the way in which the scale of choice models needs to be adjusted when different data sets are pooled (see e.g. Ben-Akiva and Morikawa 1990; Ben-Akiva

[6] But note that in our situation, smaller choice sets may include alternatives that are not present in the larger choice set, so that there is no actual sampling process.

et al. 1994). Further research is needed to further explore this method in the context of data sets where choice sets are relatively small, or the choice set compositions are systematic. However, initial results obtained in the context of the Swiss Metro dataset (Antonini et al. 2007) appear promising (see Appendix B).

Appendix A

This appendix derives an exact correction factor to account for variation in the choice set size in the μRRM model and its special cases under a very stylized situation; namely, one in which choice sets consist of exact replicates of one another. While it is very clear that this stylized situation is unlikely to occur in real life, it provides support for the notion that the correction factor also works well in the context of more realistic choice situations.

More formally, suppose that a data set consists of observations having choice set C_1 and choice observations having choice set C_2. Choice set 1 $C_1 = \{a, b, ..., z\}$ consists of Z alternatives, and choice set *2* $C_2 = \{C_1, C_1, C_1 C_1, ..., C_1\}$ consists of L exact replications of set C_1.

Theorem: *To account for variation in the choice set size in the μRRM model and its special cases (the P-RRM model and the classical RRM model) such that the ratio of choice probabilities of any two alternatives a and b is constant, regardless of the choice set size*[7] *(i.e. regardless of the number of replications of* C_1*), regret levels need to be scaled with a factor* Γ / Z_n*, where* Z_n *denotes the choice set size of choice observation n, and* Γ *is a constant.*

Proof:

The ratios of the choice probabilities of alternatives *a* and *b* in choice set C_1 and choice set C_2 are given in equation A.1:

$$\frac{P(a|C_1)}{P(b|C_1)} = \frac{e^{-\mu \sum\limits_{\substack{j \neq a \\ j \in C_1}} \sum\limits_m \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{jm} - x_{am}\right]\right)\right)}}{e^{-\mu \sum\limits_{\substack{j \neq b \\ j \in C_1}} \sum\limits_m \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{jm} - x_{bm}\right]\right)\right)}}, \qquad \frac{P(a|C_2)}{P(b|C_2)} = \frac{e^{-\mu \sum\limits_{\substack{j \neq a \\ j \in C_2}} \sum\limits_m \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{jm} - x_{am}\right]\right)\right)}}{e^{-\mu \sum\limits_{\substack{j \neq b \\ j \in C_2}} \sum\limits_m \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{jm} - x_{bm}\right]\right)\right)}} \tag{A.1}$$

Acknowledging that C_1 is a proper subset of C_2 we can rewrite the ratio of the choice probabilities of alternative *a* and *b* in choice set C_2 using C_1 (equation A.2).

$$\frac{P(a|C_2)}{P(b|C_2)} = \frac{e^{-\mu\left[L\cdot\left(\sum\limits_{\substack{j \neq a \\ j \in C_1}} \sum\limits_m \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{jm} - x_{am}\right]\right)\right)\right) - (L-1)\cdot\ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{am} - x_{am}\right]\right)\right)\right]}}{e^{-\mu\left[L\cdot\left(\sum\limits_{\substack{j \neq b \\ j \in C_1}} \sum\limits_m \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{jm} - x_{bm}\right]\right)\right)\right) - (L-1)\cdot\ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{bm} - x_{bm}\right]\right)\right)\right]}} \tag{A.2}$$

Noting that $\ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{am} - x_{am}\right]\right)\right) = \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{bm} - x_{bm}\right]\right)\right) = \ln(2)$, and that constants are irrelevant in discrete choice models, equation A.2 reduces to equation A.3.

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[7] Note that this stylized situation is a variation on the "uniform expansions of the choice set axiom" (Yellott 1977). Yellot (1977) showed that this axiom implied that a discrete choice model with i.i.d. error terms is a multinomial logit model.

$$\frac{P(a \mid C_2)}{P(b \mid C_2)} = \frac{e^{-\mu\left[L \cdot \sum_{\substack{j \neq a \\ j \in C_1}} \sum_m \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{jm} - x_{am}\right]\right)\right)\right]}}{e^{-\mu\left[L \cdot \sum_{\substack{j \neq b \\ j \in C_1}} \sum_m \ln\left(1+\exp\left(\frac{\beta_m}{\mu}\left[x_{jm} - x_{bm}\right]\right)\right)\right]}} \quad \text{A.3}$$

From equation A.3 it is easily seen that to ensure that $\frac{P(a \mid C_1)}{P(b \mid C_1)} = \frac{P(a \mid C_2)}{P(b \mid C_2)}$ holds, regret levels in A.3 need to be divided by L. Without this division, the ratio becomes much larger for set C_2, and more so when L increases, implying that the choice probability for the least (most) attractive alternative approaches 0 (1) as L becomes very large.

In practical applications, choice sets are almost never exact replications of one another. Therefore, instead of using the number of replications L of a given choice set to correct the regret levels, the final step is to write the correction factor as a function of the choice set size. The choice set sizes of C_1 and C_2 are respectively Z and L x Z. L cancels out in equation A.3. This implies that to ensure that $\frac{P(a \mid C_1)}{P(b \mid C_1)} = \frac{P(a \mid C_2)}{P(b \mid C_2)}$ holds, regret levels in A.1 and A.3 need to be scaled with a factor Γ / Z_n, where Z_n denotes the choice set size of choice observation n, and Γ is a constant. Under the parameterization $J_n = Z_n$ this correction factor is equal to the correction factor in equation 2.

Finally, it is important to mention once again that in principle the size of Γ is free to choose by the choice modeller (e.g. set to one). However, it is important to acknowledge that Γ is confounded with the scale parameter μ. As a result of that, in the some RRM models (more specifically, in the classical RRM and the G-RRM model) the choice of Γ is consequential for the imposed behaviour.
Q.E.D.

Appendix B

This appendix explores a method that can be used to account for variation in choice set size in the context of small choice sets, or when the composition of the choice set varies systematically across observations. This method involves estimating *choice set size specific correction parameters*, denoted Λ, and is akin to the way in which the scale of choice models needs to be adjusted when different data sets are pooled (see e.g. Ben-Akiva and Morikawa 1990; Ben-Akiva et al. 1994).

Data

To explore this method we use Stated Preference mode choice data collected in Switzerland in 1998. In this experiment respondents were presented choice sets consisting of either two or three alternatives: a conventional train alternative, a Swiss metro alternative (a mag-lev underground system), and a car alternative available only to car owners. Out of the total 6768 choice observations, 1131 have choice set size 2 and the remaining 5607 have choice set size 3. Alternatives are defined in terms of two attributes: travel cost and travel time. See Antonini et al. (2007) for a more detailed discussion on the data set. Finally, note that the choice set composition in these data is highly systematic. Therefore, the method proposed in the main text of this paper can be expected to perform less well for these data.

Model specification

We estimated three models: (1) a μRRM model without correction factors, and (2) a μRRM model with choice set size specific correction factors, and (3) a linear-additive RUM model, see Table B.1. In order to identify model 2, one of the choice set size specific correction factors needs to be fixed. Therefore, we fix the correction factor associated with choice set of size 2, $\Lambda_{J=2}$, to one, and estimate the correction factor associated with choice set of size 3: $\Lambda_{J=3}$ (jointly with the models' taste parameters).

Model		Regret / Utility function		Choice probability
1	μRRM model without correction factors	$R_i = ASC_i + \sum_{j \neq i} \sum_m 1 + e^{\frac{\beta_m}{\mu}\left[x_{jm} - x_{im}\right]}$		$P_i = \frac{e^{-\mu R_i}}{\sum_J e^{-\mu R_j}}$
2	μRRM model without correction factors	*J*=2	$R_i = \Lambda_{J=2}\left(ASC_i + \sum_{j \neq i} \sum_m 1 + e^{\frac{\beta_m}{\mu}\left[x_{jm} - x_{im}\right]}\right)$	$P_i = \frac{e^{-\mu R_i}}{\sum_J e^{-\mu R_j}}$
		J=3	$R_i = \Lambda_{J=3}\left(ASC_i + \sum_{j \neq i} \sum_m 1 + e^{\frac{\beta_m}{\mu}\left[x_{jm} - x_{im}\right]}\right)$	
3	Linear-additive RUM	$V_i = ASC_i + \beta_{TIME} x_{i\,TIME} + \beta_{COST} x_{i\,COST}$		$P_i = \frac{e^{V_i}}{\sum_J e^{V_j}}$

Table B.1: Models, regret and utility specifications and associated choice probability formulae

Results

Table B.2 shows the estimation results. Three key observations can be made. Firstly, in terms of model fit, the results show that both μRRM models substantially outperform the RUM model. Secondly, looking more closely at the μRRM models, we see a very significant increase in log-likelihood for the μRRM model with the choice set size specific correction factors as compared to the model without correction factors. Thirdly, the estimated choice set specific correction factor $\Lambda_{J=3}$ is considerably larger than one: $\Lambda_{J=3} = 3.60$. This indicates that the correction factor discussed in the main text of the paper – which would impose a correction factor of 2/3 in this case – would result in inferior empirical performance, probably even worse than a naïve approach without a correction factor.

Swiss Metro data

MODEL	(1) µRRM-MNL Without correction factor			(2) µRRM-MNL With choice set size specific correction factor			(3) RUM-MNL		
Number of observations	6768			6768			6768		
Null Log-likelihood	-6964.7			-6964.7			-6964.7		
Final Log-likelihood	-5264.9			-5145.8			-5331.3		
ρ^2	0.244			0.261			0.235		
	Est	*SE*	*t-stat*	*Est*	*SE*	*t-stat*	*Est*	*SE*	*t-stat*
Alternative Specific Constants									
Car	0.00	--fixed--		0.00	--fixed--		0.00	--fixed--	
SM	-0.06	0.030	-1.92	-0.21	0.07	-3.08	0.16	0.043	3.58
Train	0.29	0.088	3.30	0.75	0.20	3.77	-0.55	0.046	-11.85
Taste parameters									
β_{COST}	-0.76	0.036	-21.08	-0.22	0.03	-7.14	-1.08	0.052	-20.91
β_{TIME}	-0.99	0.042	-23.53	-0.25	0.04	-6.89	-1.28	0.057	-22.46
Correction and scale parameters									
$\Lambda_{J=2}$				1.00	--fixed--				
$\Lambda_{J=3}$				3.60	0.48	7.51			
μ	1.87	0.548	3.41	0.34	0.10	3.39			

Table B.2: Estimation results

Finally, it is important to point out that by estimating choice set size specific correction parameters (model 2) also heteroskedasticity across choice sets of different size may be picked up by the model. To investigate the extent to which such heteroskedasticity may have been captured, we analyse the extent to which the regret minimizing behaviour imposed by model 2 is different between the choice sets of size 2 and 3. To do so, we use the recently proposed measure of profundity of regret (equation B.1), see Van Cranenburgh et al. (2015) for more details. Table B.3. shows the profundity of regret associated with travel cost and travel time. It reveals that the imposed behaviour is not the same across the choice set sizes: the profundity of regret associated with travel time is substantially higher in choice sets of size 3 than in choice sets of size 2. This suggests that the model has captured at least some heteroskedasticity across choice sets of different size by accounting for variation in choice set size using choice set size specific correction parameters on these data.

$$\alpha_m = \frac{1}{|A_m|}\sum_{A_m}\left(\frac{e^{\frac{\beta_m}{\mu}\left[x_{jmn}-x_{imn}\right]}-1}{e^{\frac{\beta_m}{\mu}\left[x_{jmn}-x_{imn}\right]}+1}\right) \quad \text{where } A_m = \left\{x_{jmn}-x_{imn} \mid x_{jmn}-x_{imn} \neq 0\right\} \qquad \text{B.1}$$

	Choice set size	
	J = 2	J = 3
α_{COST}	0.24	0.23
α_{TIME}	0.06	0.12

Table B.3 Profundity of regret

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